The Korteweg–de Vries (KdV) equation,
\[ u_t + u_{xxx} - 6uu_x = 0, \quad x \in \mathbb{R}, \quad t \geq 0, \]
is a non-linear dispersive equation describing shallow water waves and possessing many intriguing properties. One of them is existence of the so-called soliton solutions [5] representing solitary waves travelling with constant speed and shape, as well as a special way in which several such solitons interact. Another interesting fact is that solutions of the KdV can be obtained as solutions of the inverse scattering problem for the family of associated Schrödinger operators [1], and the soliton solutions of the KdV correspond precisely to the so-called reflectionless potentials [4].

The aim of this talk is two-fold. Firstly, we characterise the family of all Schrödinger operators with integrable reflectionless potentials and give an explicit formula producing all such potentials. Secondly, we use the inverse scattering transform approach to describe all solutions of the KdV equation whose initial \((t = 0)\) profile is an integrable reflectionless potential. Such solutions will stay integrable and reflectionless for all \(t \geq 0\) and can be called generalized soliton solutions of KdV.

This research extends and specifies in several ways the previous work on reflectionless potentials [3, 6] and generalized soliton solutions of the KdV equation [2, 6].

**References**


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